A Study of Information Theory and Communication Characteristics of Fading Channels

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Abstract- For the transmission of signals in wireless communication, fading is considered to be the most important problem. In this paper we have analyzed the information theory and communication aspects of fading channels. For such purpose we have discussed the commonly used mathematical models of fading channels. Channel capacity plays an important role in information theory. The channel capacity is the amount of information that can be reliably transmitted over a communication channel.

Keywords – Wireless communication, fading channels, information theory, capacity

I. INTRODUCTION

There have been some recent developments in the field of wireless communication for the study of fading channels (basically for Gaussian dispersive channels) [1]-[8], [54]. The increasing need of wireless communication makes it important to evaluate the capacity limits of fading channels. The channel capacity is the amount of information to be transferred reliably over a communication channel. The information theory developed by Claude E. Shannon provides an approach towards channel capacity and the way to compute it through a mathematical model [9]. Several state-of-the-art coding systems provides good details about the information theoretic features of the communication system like the space–time codes, which undertakes the benefits of the enhanced capacity of spatial diversity in transmission and reception, i.e., multiple transmit and receive antennas [10]-[14]. The information theoretic feature has also influenced the turbo-coded multilevel modulation schemes [15] and the bit interleaved coded modulation (BICM) [16], as a special case, illustrating the capacity limit performance in the Gaussian and fading channels. We can see the examples of information theory as delay limited capacity [17], [18], the polymatroidal property of the multiple user capacity area [19], capacity versus outage [20], generalized random TDMA accessing [21], etc. The study and developments in the field of information theory has led to the beneficial development in the field of wireless communication and has reduced the effect of fading.

The rest of the paper is sectioned as follows-In Section II, we have discussed about the channel model. In Section III, we have discussed about the information theoretic features of communication using fading channels. In Section IV, the conclusion is made.

II. CHANNEL MODEL

The mathematical model for the multipath fading channels are detailed in [6], [22], and [23]. But in this section we have given a brief review of the commonly used mathematical models multipath fading channels.

A. The Scattering Function of the Channel

Characterization of multipath fading channel is done through a time-variant system. The received bandpass signal is given as-

\[ x(t) = \sum_n A_n(t)s(t - \tau_n(t)) \]  

(1)

where \( s(t) \) is the transmitted signal, \( A_n(t) \) is the attenuation factor for the signal received on the \( n^{th} \) path and \( \tau_n(t) \) is the propagation delay for the \( n^{th} \) path. \( s(t) \) can be given as-
s(t) = Re[s_{ip}(t)e^{j2\pi f_ct}] \tag{2}

where \(s_{ip}(t)\) is the equivalent lowpass transmitted signal. Substitute (2) into (1) then

\[x(t) = \text{Re}\left[\sum_n A_n(t)e^{-j2\pi f_c\tau_n t}s_{ip}(t - \tau_n(t))\right]e^{j2\pi f_ct}\]

It is clear from equation (3) that equivalent lowpass received signal is-

\[r_{ip}(t) = \sum_n A_n(t)e^{-j2\pi f_c\tau_n t}s_{ip}(t - \tau_n(t))\]

It shows that equivalent low pass channel is explained by the time-variant impulse response

\[c(\tau) = \sum_n A_n(t)e^{-j2\pi f_c\tau_n t}\delta(\tau - \tau_n(t))\]

\(c(\tau, t)\) denotes the response of the channel at time \(t\) due to an impulse applied at time \(t - \tau\). (Assuming that \(c(\tau, t)\) is wide-sense-stationary (WSS))

The time-variant transfer function \(C(f; t)\) can be defined as the Fourier transform of \(c(\tau, t)\) i.e.,

\[C(f; t) = \int_{-\infty}^{\infty} c(\tau, t)e^{-j2\pi ft}d\tau\]

Here consideration is made that the multipath signals propagating through the channel at different delays are uncorrelated (a wide-sense stationary uncorrelated scattering, or WSSUS, channel). So the scattering function \(S(\tau; \lambda)\) describes a doubly spread channel, which is a measure of the power spectrum of the channel at delay \(\tau\) and frequency offset \(\lambda\) (relative to the carrier frequency). The delay power spectrum of the channel (also called the multipath intensity profile) can be calculated by simply averaging \(S(\tau; \lambda)\) over \(\lambda\), i.e.,

\[S_d(\tau) = \int_{-\infty}^{\infty} S(\tau; \lambda)d\lambda\]

Similarly, the Doppler power spectrum is

\[S_d(\lambda) = \int_{-\infty}^{\infty} S(\tau; \lambda)d\tau\]

The range of values over which the delay power spectrum \(S_d(\tau)\) is nonzero is known as the multipath spread \(T_m\) of the channel. In the similar manner, the range of values over which the Doppler power spectrum \(S_d(\lambda)\) is nonzero, is known as the Doppler spread \((B.W)_d\) of the channel.

The Doppler spread \((B.W)_d\) provides a measure of how rapidly the channel impulse response varies in time. The larger the value of \((B.W)_d\), the more rapidly the channel impulse response is changing with time. So this provides an another channel parameter, known to be the channel coherence time \(T_{cc}\) as-

\[T_{cc} = \frac{1}{(B.W)_d}\]

Slow fading channel is one in which the coherence time is larger than the symbol period while fast fading channel provides the smaller coherence time in comparison to the symbol period.

Similarly, the channel coherence bandwidth \(B_{cc}\) is the reciprocal of the multipath spread, i.e.,

\[B_{cc} = \frac{1}{T_m}\]

B. The Multipath model for the Frequency Nonselective Channel

From equation (10) we can say that if \(B_{cc}\) is larger than the bandwidth of the transmitted signal, then the channel is known as frequency nonselective channel. Such type of fading channels has a time varying multiplicative effect on the signal transmitted and these multipath components of the channel cannot be resolved.

If the signal transmitted has smaller time duration compared to the coherence time of the channel, this frequency selective channel is termed as slow fading channel. On the other hand, in the vice-versa condition i.e. when the transmitted signal has time duration greater than the coherence time, this channel is termed as fast fading channel.
C. The Tapped Delay Line model for the Frequency selective Channel

From equation (10) we can say that if $B_{cc}$ is smaller than the bandwidth of the transmitted signal, then the channel is known as frequency selective channel. Additional distortion is caused by the time variations in $C(t; f)$, which is the fading effect i.e. a time variation in the received signal strength of the frequency components in $X(f)$, the frequency component of $x(t)$.

The channel model consists of a tapped delay line with uniformly spaced taps. The tap spacing between adjacent taps is $\frac{1}{W}$ where $W$ is the bandwidth of the signal transmitted over the channel. The tap coefficients, represented as $c_n(t) = A_n(t)e^{j\theta_n t}$, are usually modelled as complex-valued Gaussian random processes that are mutually uncorrelated. The length of the delay line corresponds to the multipath spread i.e.,

$$T_m = \frac{N}{W}$$  \hspace{1cm} (11)

![Diagram of the Tapped Delay Line model for the Frequency selective Channel](image)

In equation (11), where $N$ represents the maximum number of possible multipath signal components.

III. INFORMATION THEORY FEATURES

Some assumptions are made in information theoretic aspect for fading channels as the understanding of the full promise of diversity systems, mainly transmitter diversity. Other information-theoretic measures in fading channel as error exponents and cutoff rates will only be considered briefly. The idea for the change in fading
process during the transmitted block depicting information-theoretic arguments will be considered, focussing the ergodic capacity, distribution of capacity (giving rise to the “capacity-versus-outage” approach) and delay-limited capacity approach.

Multiple-user systems involve some technologies and accessing protocols like code-division multiple access (CDMA), time-division multiple access (TDMA), frequency-division multiple access (FDMA), successive cancellation [2], rate splitting [24], and L-out-of-K models [25] in relation with the fading channel. Delay limited capacity regions using compound channel for the fading models have also been considered in the past. Cellular fading models [26], [27], [28], and [29], have been considered for such purpose. In [30], Wyner’s model is considered and its fading variants are considered in [31] and [32]. These are centred at the information theoretic feature of the channel accessing inter-and intracell protocol like CDMA and TDMA.

For the single user systems we can define as-

Assuming the channel with channel input $x_n \in \mathcal{X}$ and output $y_n \in \mathcal{Y}$ and state $s_n \in \mathcal{S}$. Here $\mathcal{X}, \mathcal{Y}, \mathcal{S}$ represents the respective spaces. The channel states specify a conditional distribution $\{p(y|x,s), s \in \mathcal{S}\}$ where the channel is considered to be memoryless.

$$p(Y^n|X^n, S^n) = \prod_{n=1}^{n} p(Y_n|x_n, s_n) \quad (12)$$

Figure 2. Block diagram of the channel with time-varying state and transmitter and receiver CSI

The channel state information is provided to the transmitter and receiver, represented by $u \in \mathcal{U}$ and $v \in \mathcal{V}$, via some conditional memoryless distribution

$$p(u^n|x^n, s^n) = \prod_{n=1}^{n} p(u_n|v_n, s_n) \quad (13)$$

The channel capacity has been provided by Shannon [261] as-

$$C = \max_{q(t)} I(\mathcal{T}, Y) \quad (14)$$

where $\mathcal{T} = \{x_1, \ldots, x_d\}$ is a random input vector of length whose value is equal to the cardinality $|\mathcal{S}|$ of $\mathcal{S}$ with elements in $\mathcal{X}$, where $q(t)$ is the probability distribution of $\mathcal{T}$. In the ergodic capacity, the basic consideration is that $T \gg T_{ee} = \frac{1}{(Wd)}$. It means that the transmission line is so long as to long term ergodic characteristics of the fading process $c(r; t)$ and it is a ergodic process in $t$.

So for this case, a majority of references are given [20], [21], [33]-[39], [40] and [41]. In [42] and [43] we can see that at rates lower than capacity, the error probability is exponentially decaying with the transmission length.

It should be kept in mind that, under perfect knowledge of CSI at the receiver, the details of whether the fading varies on a continuous or block-fading fashion are unvalued in terms of the ergodic capacity; for any ergodic fading process, the capacity is a function only of the stationary distribution of the fading, irrespective of its correlation [56].

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The capacity versus outage (capacity distribution) performance is determined by the probability that the channel cannot support a given rate. This means that outage probability is associated with any given rate. This is a standard method.

The Delay limited capacities are given by the zero outage probabilities. Any positive rate that corresponds to zero outage will result in a positive delay limited capacity [18]. In [17] the delay-limited capacity is connected with the reliable transmitted rate which is invariant and independent of the actual realization of the fading random phenomenon. In [33] the condition for the positive delay-limited capacity with noisy CSI has been described for both the transmitter and receiver. Simple buffer control policies have been characterized (which exhibit optimal characteristics) in connection to the delay-limited capacity and the expected capacity of fading channels [53].

Diversity is a most important information theoretic measure. It plays an important role in handling the harmful effects of fading and time-varying features of the channel. In [44], we have studied that the transmitter diversity provides an abundant increment of the achievable rates which is creating a great scope. But at the receiver’s end the space diversity is common in practice. In [20] receiver diversity with CSI is available. In this capacity along with its distributions are explained for two diversity branches with optimal (maximal-gain) or suboptimal (selection) combining correlating both diversity branches. It was determined that the useful effect of diversity destroys only at very high correlations. Receiver Capacity with CSI for Ricean as well and Nakagami-m distributions with independent diversity reception is evaluated in [40] and [37]. For the same reason capacity close to Gaussian was confirmed for the moderate degrees of diversity.

Considering the error exponents and cutoff rates we can see in [4] that the standard random coding error exponent serving as a lower bound on the optimal error exponent and the sphere-packing upper bound corresponds to the rates larger than the critical rate resulting in the correct exponential behavior for these classes of channels. The cutoff rate [45], calculating both an achievable rate and the magnitude of the random-coding error exponent, provides another interesting information-theoretic measure. Error exponents for fading channels have been undertaken in [46] and [47] for various cases; in [47] the unknown CSI has been considered. In [48], the error exponent for the case of infinite bandwidth but finite power has been determined for the no-CSI environment. In [49] and [50] where exact capacity and error exponent are considered respectively for the performance of the fading channel which is being measured per-unit cost (power). The random coding error has been examined in [51] for multiple antennas at transmitter and receiver ends and for the block fading channel.

In [52], the random-coding error exponent for a single-dimensional fading channel is determined with ideal CSI available to the receiver, and the corresponding error exponent for Gaussian-distributed inputs is considered in the region above the critical rate through capacity. There are many more references providing such measures’ details.

IV. CONCLUSION

Here in this paper we have tried to review some information theoretic aspects of digital communication over fading channels. Here does not the study stop. There are various papers which provide the vast knowledge of such topics. In this work, we have considered that the transmitter has knowledge channel state information. But it can be that in other cases like transmitter and receiver both do not have CSI. Finally, we have only assumed single-user channels while multiple-users models can also be considered. The reference list provided here is also not sufficient, it can be extended further. This paper provides the subjective details to some extent.

V. REFERENCE


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